# Matemáticas II.
# Curso 2012-2013.
# Problemas de integral indefinida

Calcular las siguientes integrales:

1. $\displaystyle\int \frac{1+x^2}{\sqrt{x}}\,\mathrm{d}x$
2. $\displaystyle\int \frac{x^2+2x}{(x+1)^2}\,\mathrm{d}x$
3. $\displaystyle\int \cos 3x\,\mathrm{d}x$
4. $\displaystyle\int \frac{\operatorname{sen} x}{\cos^2 x}\,\mathrm{d}x$
5. $\displaystyle\int \frac{1}{1+\cos x}\,\mathrm{d}x$
   (multiplicar y dividir por $1-\cos x$)
6. $\displaystyle\int (\operatorname{tg} 2x + \sec 2x)^2\,\mathrm{d}x$
7. $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$
8. $\displaystyle\int \frac{1}{9+x^2}\,\mathrm{d}x$
9. $\displaystyle\int \frac{1}{\sqrt{25-16x^2}}\,\mathrm{d}x$
10. $\displaystyle\int \frac{1}{4x^2+9}\,\mathrm{d}x$
11. $\displaystyle\int \frac{1}{x\sqrt{4x^2-9}}\,\mathrm{d}x$
12. $\displaystyle\int \frac{x^2}{\sqrt{1-x^6}}\,\mathrm{d}x$
    (hacer $t = x^3$)
13. $\displaystyle\int \frac{x}{x^4+3}\,\mathrm{d}x$
    (hacer $t = x^2$)
14. $\displaystyle\int \frac{1}{x\sqrt{x^4-1}}\,\mathrm{d}x$
15. $\displaystyle\int \frac{3x^3-4x^2+3x}{x^2+1}\,\mathrm{d}x$
16. $\displaystyle\int \frac{\sec x \operatorname{tg} x}{9+4\sec^2 x}\,\mathrm{d}x$
17. $\displaystyle\int \frac{x+3}{\sqrt{1-x^2}}\,\mathrm{d}x$
18. $\displaystyle\int \frac{1}{x^2+10x+30}\,\mathrm{d}x$
19. $\displaystyle\int \frac{1}{\sqrt{20+8x-x^2}}\,\mathrm{d}x$
20. $\displaystyle\int \frac{1}{2x^2+2x+5}\,\mathrm{d}x$
21. $\displaystyle\int \frac{1}{2\sqrt{28-12x-x^2}}\,\mathrm{d}x$
22. $\displaystyle\int \frac{1}{\sqrt{5-4x-x^2}}\,\mathrm{d}x$
23. $\displaystyle\int \frac{x+2}{\sqrt{4-x^2}}\,\mathrm{d}x$
24. $\displaystyle\int (x-2)^{\frac{3}{2}}\,\mathrm{d}x$
25. $\displaystyle\int \frac{1}{(x-1)^3}\,\mathrm{d}x$
26. $\displaystyle\int \frac{1}{\sqrt{x+3}}\,\mathrm{d}x$
27. $\displaystyle\int \sqrt{3x-1}\,\mathrm{d}x$
28. $\displaystyle\int \sqrt{2-3x}\,\mathrm{d}x$
29. $\displaystyle\int (2x^2+3)^{\frac{1}{3}}\,\mathrm{d}x$
30. $\displaystyle\int \sqrt{1+x^4}\,x^3\,\mathrm{d}x$
31. $\displaystyle\int \frac{x}{(x^2+4)^3}\,\mathrm{d}x$
32. $\displaystyle\int (x-1)^2 x\,\mathrm{d}x$
33. $\displaystyle\int (x^2-x)^4(2x-1)\,\mathrm{d}x$
34. $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x-4}}\,\mathrm{d}x$
35. $\displaystyle\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,\mathrm{d}x$
36. $\displaystyle\int \frac{(x+1)(x-2)}{\sqrt{x}}\,\mathrm{d}x$
37. $\displaystyle\int \sec 3x \operatorname{tg} 3x\,\mathrm{d}x$
38. $\displaystyle\int \operatorname{cosec}^2 2x\,\mathrm{d}x$
39. $\displaystyle\int x\sec^2 x^2\,\mathrm{d}x$
40. $\displaystyle\int \operatorname{tg}^2 x\,\mathrm{d}x$
41. $\displaystyle\int \cos^4 x \operatorname{sen} x\,\mathrm{d}x$
42. $\displaystyle\int \frac{1}{5-x^2}\,\mathrm{d}x$
43. $\displaystyle\int \frac{\sec^2 x}{1-4\operatorname{tg}^2 x}\,\mathrm{d}x$

Soluciones (no se ha puesto la constante de integración):

(1) $2\sqrt{x}\,(1+\frac{2}{3}x+\frac{1}{5}x^2)$
(2) $\frac{x^2}{x+1}$
(3) $\frac{1}{3}\operatorname{sen} 3x$
(4) $\sec x$
(5) $-\operatorname{cotg} x+\operatorname{cosec} x$
(6) $\operatorname{tg} 2x+\sec 2x-x$
(7) $\operatorname{arsen}\frac{x}{2}$
(8) $\frac{1}{3}\operatorname{artg}\frac{x}{3}$
(9) $\frac{1}{4}\operatorname{arsen}\frac{4x}{5}$
(10) $\frac{1}{6}\operatorname{artg}\frac{2x}{3}$
(11) $\frac{1}{3}\operatorname{arsec}\frac{2x}{3}$
(12) $\frac{1}{3}\operatorname{arsen} x^3$
(13) $\frac{\sqrt{3}}{6}\operatorname{artg}\frac{x^2\sqrt{3}}{3}$
(14) $\frac{1}{2}$
(15) $\frac{3x^2}{2}-4x+4\operatorname{artg} x$
(16) $\frac{1}{6}\operatorname{artg}\frac{2\sec x}{3}$
(17) $-\sqrt{1-x^2}+3\operatorname{arsen} x$
(18) $\frac{\sqrt{5}}{5}\operatorname{artg}\frac{(x+5)\sqrt{5}}{5}$
(19) $\operatorname{arsen}\frac{x-4}{6}$
(20) $\frac{1}{3}\operatorname{artg}\frac{2x+1}{3}$
(21) $\operatorname{arsen}\frac{x+6}{8}$
(22) $-\sqrt{5-4x-x^2}+\operatorname{arsen}\frac{x+2}{3}$
(23) $-\sqrt{4x-x^2}+4\operatorname{arsen}\frac{x-2}{2}$
(24) $\frac{2}{5}(x-2)^{\frac{5}{2}}$
(25) $-\frac{1}{2(x-1)^2}$
(26) $2\sqrt{x+3}$
(27) $\frac{2}{9}(3x-1)^{\frac{3}{2}}$
(28) $-\frac{2}{9}(2-3x)^{\frac{3}{2}}$
(29) $\frac{3}{16}(2x^2+3)^{\frac{4}{3}}$
(30) $\frac{1}{6}(1+x^4)^{\frac{3}{2}}$
(31) $-\frac{1}{4(x^2+4)^2}$
(32) $\frac{1}{4}x^4-\frac{2}{3}x^3+\frac{1}{2}x^2$
(33) $\frac{1}{5}(x^2-x)^5$
(34) $\sqrt{x^2+2x-4}$
(35) $\frac{2}{3}(1+\sqrt{x})^3$
(36) $\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{3}x^{\frac{3}{2}}-4x^{\frac{1}{2}}$
(37) $\frac{1}{3}\sec 3x$
(38) $-\frac{1}{2}\operatorname{cotg} 2x$
(39) $\frac{1}{2}\operatorname{tg} x^2$
(40) $\operatorname{tg} x-x$
(41) $-\frac{1}{5}\cos^5 x$
(42) $\operatorname{arsen}\frac{x\sqrt{5}}{5}$
(43) $\frac{1}{2}\operatorname{arsen}(2\operatorname{tg} x)$

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44. $\displaystyle\int 3^{2x}\,\mathrm{d}x$
45. $\displaystyle\int \frac{e^{\frac{1}{x}}}{x^2}\,\mathrm{d}x$
46. $\displaystyle\int (e^x+1)^3e^x\,\mathrm{d}x$
47. $\displaystyle\int \frac{\mathrm{d}x}{e^x+1}$
48. $\displaystyle\int \frac{e^{\frac{1}{x^2}}}{x^3}\,\mathrm{d}x$
49. $\displaystyle\int e^{-x^2+2}x\,\mathrm{d}x$
50. $\displaystyle\int (e^x+1)^2\,\mathrm{d}x$
51. $\displaystyle\int (e^x-x^e)\,\mathrm{d}x$
52. $\displaystyle\int \frac{e^{2x}}{e^{2x}+3}\,\mathrm{d}x$
53. $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,\mathrm{d}x$
54. $\displaystyle\int x^3\,5^{x^4+1}\,\mathrm{d}x$

Soluciones (no se ha puesto la constante de integración):

(44) $\frac{1}{2\ln 3}3^{2x}$
(45) $-e^{\frac{1}{x}}$
(46) $\frac{(e^x+1)^4}{4}$
(47) $x-\ln(e^x+1)$
(48) $-\frac{1}{2}e^{\frac{1}{x^2}}$
(49) $-\frac{1}{2}e^{-x^2+2}$
(50) $\frac{1}{2}e^{2x}+2e^x+x$
(51) $e^x-\frac{x^{e+1}}{e+1}$
(52) $\frac{1}{2}\ln(e^{2x}+3)$
(53) $\operatorname{arsen} e^x$
(54) $\frac{1}{4\ln 5}5^{x^4+1}$

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55. $\displaystyle\int x^3e^{x^2}\,\mathrm{d}x$
56. $\displaystyle\int \ln(x^2+2)\,\mathrm{d}x$
57. $\displaystyle\int \ln x\,\mathrm{d}x$
58. $\displaystyle\int x\operatorname{sen} x\,\mathrm{d}x$
59. $\displaystyle\int x^2\ln x\,\mathrm{d}x$
60. $\displaystyle\int \operatorname{arsen} x\,\mathrm{d}x$
61. $\displaystyle\int \operatorname{artg} x\,\mathrm{d}x$
62. $\displaystyle\int \sec^3 x\,\mathrm{d}x$
63. $\displaystyle\int \arccos 2x\,\mathrm{d}x$

64. $\displaystyle\int x^2 \operatorname{sen} x \, \mathrm{d}x$

65. $\displaystyle\int x^3 e^{2x} \, \mathrm{d}x$

66. $\displaystyle\int x \cos x \, \mathrm{d}x$

67. $\displaystyle\int x \sec^2 3x \, \mathrm{d}x$

68. $\displaystyle\int x \operatorname{artg} x \, \mathrm{d}x$

69. $\displaystyle\int x^2 e^{-3x} \, \mathrm{d}x$

70. $\displaystyle\int x^3 \operatorname{sen} x \, \mathrm{d}x$

71. $\displaystyle\int x \operatorname{arsen} x^2 \, \mathrm{d}x$

72. $\displaystyle\int \frac{\ln x}{x^2} \, \mathrm{d}x$

Soluciones (no se ha puesto la constante de integración):

(55) $\frac{1}{2}e^{x^2}(x^2-1)$
(56) $x\left(\ln(x^2+2)-2\right)+2\sqrt{2}\operatorname{artg}\frac{x}{\sqrt{2}}$
(57) $x(\ln x - 1)$
(58) $-x\cos x + \operatorname{sen} x$
(59) $\frac{x^3}{3}\ln x - \frac{1}{9}x^3$
(60) $x\operatorname{arsen} x + \sqrt{1-x^2}$
(61) $x\operatorname{artg} x - \frac{1}{2}\ln(1+x^2)$
(62) $\frac{1}{2}\left(\sec x \operatorname{tg} x + \ln|\sec x + \operatorname{tg} x|\right)$
(63) $x\arccos 2x - \frac{1}{2}\sqrt{1-4x^2}$
(64) $-x^2\cos x + 2(x\operatorname{sen} x + \cos x)$
(65) $\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x}$
(66) $x\operatorname{sen} x + \cos x$
(67) $\frac{1}{3}x\operatorname{tg} 3x - \frac{1}{9}\ln|\sec x|$
(68) $\frac{1}{2}(x^2+1)\operatorname{artg} x - \frac{1}{2}x$
(69) $-\frac{1}{3}e^{-3x}\left(x^2+\frac{2}{3}x+\frac{2}{9}\right)$
(70) $-x^3\cos x + 3x^2\operatorname{sen} x + 6x\cos x - 6\operatorname{sen} x$
(71) $\frac{1}{2}x^2\operatorname{arsen} x^2 + \frac{1}{2}\sqrt{1-x^2}$
(72) $-\frac{\ln x+1}{x}$

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73. Demostrar la siguiente fórmula de reducción:

$$\int \operatorname{sen}^m x \, \mathrm{d}x = -\frac{\operatorname{sen}^{m-1} x \cos x}{m} + \frac{m-1}{m}\int \operatorname{sen}^{m-2} x \, \mathrm{d}x$$

74. Aplicar la fórmula anterior para calcular la integral de $\operatorname{sen}^2 x$.

75. Calcular la integral de $\operatorname{sen}^3 x$.

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76. $\displaystyle\int \frac{1}{x^2-9} \, \mathrm{d}x$

77. $\displaystyle\int \frac{x}{x^2-3x-4} \, \mathrm{d}x$

78. $\displaystyle\int \frac{x^2+3x-4}{x^2-2x-8} \, \mathrm{d}x$

79. $\displaystyle\int \frac{x}{(x-2)^2} \, \mathrm{d}x$

80. $\displaystyle\int \frac{2x+1}{x^2+x+1} \, \mathrm{d}x$

81. $\displaystyle\int \frac{3}{x^2+2x+6} \, \mathrm{d}x$

82. $\displaystyle\int \frac{x-1}{x^2+4x+5} \, \mathrm{d}x$

83. $\displaystyle\int \frac{4x-1}{2x^2+3x+2} \, \mathrm{d}x$

Soluciones (no se ha puesto la constante de integración):

(76) $\frac{1}{6}\ln\left|\frac{x-3}{x+3}\right|$
(77) $\frac{1}{5}\ln\left|(x+1)(x-4)^4\right|$
(78) $x + \ln\left|(x+2)(x-4)^4\right|$
(79) $\ln|x-2| - \frac{2}{x-2}$
(80) $\ln(x^2+x+1)$
(81) $\frac{3}{\sqrt{5}}\operatorname{artg}\frac{x+1}{\sqrt{5}}$
(82) $\frac{1}{2}\ln(x^2+4x+5) - 3\operatorname{artg}(x+2)$
(83) $\ln(2x^2+3x+2) - \frac{8}{\sqrt{191}}\operatorname{artg}\frac{4x+3}{\sqrt{191}}$

## Cálculo de algunas integrales

◇ $\displaystyle\int \sec x \, \mathrm{d}x$

$$\begin{aligned}
\int \sec x \, \mathrm{d}x &= \int \frac{1}{\cos x} \, \mathrm{d}x && t = \operatorname{sen} x \quad \mathrm{d}t = \cos x \, \mathrm{d}x \\
&= \int \frac{1}{\cos x}\frac{1}{\cos x} \, \mathrm{d}t = \int \frac{1}{\cos^2 x} \, \mathrm{d}t \\
&= \int \frac{1}{1-t^2} \, \mathrm{d}t = \frac{1}{2}\int \frac{1}{1+t} \, \mathrm{d}t + \frac{1}{2}\int \frac{1}{1-t} \, \mathrm{d}t \\
&= \frac{1}{2}\ln|1+t| - \frac{1}{2}\ln|1-t| + C \\
&= \frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C = \frac{1}{2}\ln\left|\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}\right| + C \\
&= \ln|\sec x + \operatorname{tg} x| + C
\end{aligned}$$

◇ $\displaystyle\int \sec^3 x \, \mathrm{d}x$

$$\begin{aligned}
\int \sec^3 x \, \mathrm{d}x &= \int \sec x \sec^2 x \, \mathrm{d}x && u = \sec x \, ; \quad \mathrm{d}v = \sec^2 x \, \mathrm{d}x \\
&= \sec x \operatorname{tg} x - \int \operatorname{tg} x \, \frac{\operatorname{sen} x}{\cos^2 x} \, \mathrm{d}x \\
&= \sec x \operatorname{tg} x - \int \frac{\operatorname{sen}^2 x}{\cos^3 x} \, \mathrm{d}x \\
&= \sec x \operatorname{tg} x - \int \frac{1-\cos^2 x}{\cos^3 x} \, \mathrm{d}x \\
&= \sec x \operatorname{tg} x - \int \frac{1}{\cos^3 x} \, \mathrm{d}x + \int \frac{1}{\cos x} \, \mathrm{d}x \\
&= \sec x \operatorname{tg} x - \int \sec^3 x \, \mathrm{d}x + \ln|\sec x + \operatorname{tg} x|
\end{aligned}$$

$$2\int \sec^3 x \, \mathrm{d}x = \sec x \operatorname{tg} x + \ln|\sec x + \operatorname{tg} x| + C$$

$$\int \sec^3 x \, \mathrm{d}x = \frac{1}{2}\sec x \operatorname{tg} x + \frac{1}{2}\ln|\sec x + \operatorname{tg} x| + C$$

$\diamond$ $\displaystyle\int \sqrt{1-x^2}\,\mathrm{d}x$

$$\begin{aligned}
\int \sqrt{1-x^2}\,\mathrm{d}x &= \int \sqrt{1-\operatorname{sen}^2 t}\cos t\,\mathrm{d}t \qquad\qquad x=\operatorname{sen} t\,; \quad \mathrm{d}x=\cos t\,\mathrm{d}t\\
&= \int \cos t\cos t\,\mathrm{d}t\\
&= \int \cos^2 t\,\mathrm{d}t\\
&= \frac{1}{2}\left(t+\operatorname{sen} t\cos t\right)+C\\
&= \frac{1}{2}\left(\operatorname{arsen} x + x\sqrt{1-x^2}\right)+C
\end{aligned}$$

$\diamond$ $\displaystyle\int \sqrt{x^2-1}\,\mathrm{d}x$

$$\begin{aligned}
\int \sqrt{x^2-1}\,\mathrm{d}x &= \int \sqrt{\sec^2 t-1}\,\frac{\operatorname{sen} t}{\cos^2 t}\,\mathrm{d}t \qquad\qquad x=\sec t\,; \quad \mathrm{d}x=\frac{\operatorname{sen} t}{\cos^2 t}\,\mathrm{d}t\\
&= \int \operatorname{tg} t\,\frac{\operatorname{sen} t}{\cos^2 t}\,\mathrm{d}t\\
&= \int \frac{\operatorname{sen} t}{\cos t}\,\frac{\operatorname{sen} t}{\cos^2 t}\,\mathrm{d}t\\
&= \int \frac{\operatorname{sen}^2 t}{\cos^3 t}\,\mathrm{d}t\\
&= \int \frac{1-\cos^2 t}{\cos^3 t}\,\mathrm{d}t\\
&= \int \sec^3 t\,\mathrm{d}t - \int \sec t\,\mathrm{d}t\\
&= \frac{1}{2}\sec t\operatorname{tg} t + \frac{1}{2}\ln\left|\sec t+\operatorname{tg} t\right| - \ln\left|\sec t+\operatorname{tg} t\right| + C\\
&= \frac{1}{2}\sec t\operatorname{tg} t - \frac{1}{2}\ln\left|\sec t+\operatorname{tg} t\right| + C\\
&= \frac{1}{2}\,x\sqrt{x^2-1} - \frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C
\end{aligned}$$